Diffeomorphism, kappa transformations and the theory of non-linear realisations

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ABSTRACT: We will show how the theory of non-linear realisations can be used to naturally incorporate world line diffeomorphisms and kappa transformations for the point particle and superpoint particle respectively. Similar results also hold for a general p-brane and super p-brane, however, we must in these cases include an additional Lorentz transformation.

Contents

1. Introduction

Non-linear realizations played an important role in particle physics. It has first been developed in constructing lagrangians with chiral symmetries [1]. In this paper we will consider how to use the theory of non-linear realisations to derive the dynamics and symmetries of branes. As such, it will be instructive to summarise the theory of non-linear realisations in general. For the case of internal symmetries, the original general formulation [2] of the non-linear realisation considered a group G with sub-algebra H and worked with elements of the coset G/H which depended on space-time x. Specifically, they considered coset representatives in the form of a set of elements $g(x)$ of G. Under a rigid transformation of the group g_0 the coset representatives $g(x)$ changed as $g(x) \rightarrow g_0 g(x)$. However, in order to preserve the choice of coset representatives one had to simultaneously perform a specific transformation that belonged to H and depended on g_0 and $g(x)$. Hence the coset representatives changed as

$$g(x) \rightarrow g_0 g(x) h(g_0, g(x)) \ . \tag{1.1}$$

The non-linearly realised theory is taken to be the one invariant under this symmetry transformation. To be specific it is often useful to take the group elements to be in exponential parameterisation and choose the coset representatives to be $exp(i\phi_a(x)T^a)$ where T_a are the generators of G which are not in H and $\phi_a(x)$ will become the fields of the non-linearly realised theory.

Non-linear realisations were also used [3] to derive symmetries that involved transformations of manifolds that were to be interpreted as space-times. That is the manifolds are the coset space G/H. The choice of coset representatives then provides a parameterisation of the manifold. The classic case is that of superspace [4] where the group G is the supersymmetry group involving the translation and supercharge generators, denoted generically

by P, Q and Lorentz transformations and the subgroup H is just the Lorentz group. The coset representatives may be chosen to be of the form $g(x, \theta) = exp(ixP + \bar{\theta}Q)$ where (x, θ) are interpreted as the coordinates of superspace. The transformations on superspace being given by

$$g(x, \theta) \rightarrow g_0 g(x, \theta) h(g_0, x, \theta) \tag{1.2}$$

with $h(g_0, x, \theta)$ being the corresponding compensating Lorentz transformation. The formulae for any such non-linear realisation are analogous.

We can also have non-linear realisations that concern symmetries that act on space-time and on the fields that live on it [3]. In this case we consider a group K with a sub-algebra H and a set of generators that belong to a representation L of K. Together K and L form a semi-direct product group denoted $G = K \otimes_s L$, that is the commutator of a generator of K and one of L is a generator of L. The group elements $l \in L$ will introduce space-time into theory and if they are parameterised by x these become the coordinates of the space-time which is the coset $H \otimes_s L/H$. We consider the non-linear realisation based on the coset G/H. The corresponding group element used in the non-linear realisation takes the form $k = l(x)g(x)$ where $l(x) \in L$, and $g(x)$ is a coset representative of K/H. The coset representatives $g(x)$ are taken to depend on these coordinates x and they will contain the fields that live on the space-time. Under a group transformation of $K \otimes_s L$, $g(x) \rightarrow k_0 g(x) h(k_0, l, g(x))$ where $k_0 \in G$ and the latter factor is the correcting H transformation required to preserve the choice of coset representatives in K/H. If $k_0 \in L$ then the transformation will just change the coordinates x, however, if $k_0 \in K$ then the transformation will change the coordinates x and also the fields contained in $g(x)$. One of the first examples of this type of non-linear realisation was for the conformal group in reference [5].

Although the original papers [2] of Callan, Coleman, Wess and Zumino, and much of the subsequent development, were formulated in terms of coset representatives it is more elegant to reformulate the theory of non-linear realisations in terms of the group elements of G themselves rather than just the coset G/H. We first discuss this approach [6, 7] for the case of an internal symmetry where we work with the group elements of G and take the symmetry of the non-linearly realised theory to be by definition given by

$$g(x) \rightarrow g_0 g(x) \quad \text{and} \quad g(x) \rightarrow g(x) h(x) \tag{1.3}$$

where $g_0 \in G$ is a rigid transformation while the second independent local transformation $h(x)$ is an arbitrary space-time dependent transformation that belongs to H. In this more general way of doing things the theory contains fields that can be fixed using the local H transformations. We will refer to such a formulation as a non-linear realisation of an group G with local subgroup H. One can from the beginning use these local H transformations to set to zero some fields and so work only with a coset representatives. This is equivalent to the original approach and it requires the compensating transformations discussed above. One can also work in a half way house where only some of the fields are removed. This more general approach to formulating non-linear realisation can be used for space-time symmetries as well as internal symmetries of fields. In all future discussions in this paper

we will adopt this more general way of constructing non-linear realisations although we will often use some of the local symmetry to fix certain less interesting parts of the group element.

One can view branes as solitonic objects that occur in field theories and from this perspective their motion can be seen as a non-linear realisation corresponding to the symmetries that are spontaneously broken by the soliton. The embedding coordinates transverse to the brane are the Goldstone bosons for the broken translations and in a supersymmetric theory part of the fermions correspond to the breaking of the supersymmetries by the solitons. Some early papers on the derivation of the dynamics of specific supersymmetric branes from the viewpoint of non-linear realisations are given in [8]–[12]. In general we must begin with a group K, with a specified subgroup H, together with a representation L which form a semi-direct product group denoted $G = K \otimes_s L$. We then further sub-divide the generators in the representation L of K into two sets L_1 and L_2 which are representations of H, the specified sub-algebra of K. We may think of the algebra K as an automorphism algebra of the algebra L, which is not always a commuting algebra. The local subgroup of the non-linear realisation has in the past been taken to be the group H. This procedure is most easily explained by considering the case which leads to the dynamics of the bosonic p-brane [13]. To do this we take $K = SO(1, D-1)$ and L are the translation generators in the vector representation of $K = SO(1, D-1)$. We then take L_1 to contain $p+1$ of the translation generators and $H = SO(1, p) \otimes SO(D-p-1)$. The generators of L_1 being a vector under the first factor and a scalar under the second factor of H. The sets L_1 and L_2 then contain the unbroken and broken translations and H the unbroken Lorentz rotations. The coset representatives can be generically written as

$$g = e^{ix \cdot P} e^{ix' \cdot P'} e^{i\phi \cdot J'} \tag{1.4}$$

where P, P' belong to L_1 and L_2 respectively and J' are the generators of K not in H.

Even in this apparently simple example there are still several different possible options that can be pursued. The first option concerns the Goldstone fields corresponding to broken Lorentz transformations. In reference [13] these were algebraically eliminated in terms of derivatives of the Goldstone fields for translations, using a constraint that was covariant under the non-linear realisation, that is a so called inverse Higgs constraint [14]. After this elimination an action was constructed. However, recently a new action for the bosonic p-brane was found that was a function of all the above Goldstone fields including those corresponding to broken Lorentz transformations [15]. The equations of motion for these latter Goldstone bosons were in fact non-other than the inverse Higgs constraint and using this in the action one found the previous result.

The second option is of a more fundamental nature and it concerns the way the non-linear realisation is defined from the outset. In particular, it concerns the dependence of the fields. It was usual in discussions of brane dynamics to take the unbroken translations that occurred in the group element to be associated with coordinates, which turned out to parameterise the brane world, and the Goldstone fields were taken to depend on these coordinates. However, reference [13] also introduced fields associated with the unbroken translations which appeared in the group element and these in common with all

the Goldstone fields were taken to depend on external parameters ξ that turned out to parameterise the brane world volume. It was then demanded that the theory be reparameterisation invariant as a requirement in addition to be invariant under the transformations of the non-linear realisation of equation (1.3). By choosing static gauge in the latter formulation one can recover the former formulation. Non-relativistic branes for Galilei [16] and Newton Hooke [17] groups have been constructed using non-linear realizations along the lines of the previous paragraph. The corresponding actions are Wess-Zumino terms of the previous groups.

In this paper we will consider how to incorporate the local symmetries such as world volume reparameterisation and, for supersymmetric branes, κ-symmetry into the theory of non-linear realisations. Although reference [13] had the advantage that it introduced the reparameterisation invariance in the parameters ξ to be present in brane dynamics, it also introduced fields corresponding to generators that were not broken. The resolution of this puzzle is to adopt the formulation of the theory of non-linear realisations encoded in the transformations of equation (1.3), that is work with the group elements rather than the just the coset, and take the unbroken translation generators to be part of the local subgroup of the non-linear realisation. Thus the unbroken translation generators occur in the group element with fields, which depend, like all other fields, on the parameters ξ. However, as the unbroken translations are in the local subgroup their corresponding fields can be fixed by a suitable local transformation. The transformations of the non-linear realisation corresponding to the unbroken translations should then correspond to the reparameterisation invariance of the theory. For the point particle we will find that this is indeed the case. However, for the general bosonic p-brane we will find that we must include with these local transformations an additional Lorentz rotation. In this paper will adopt the latter point of view of reference [15] in that we will find actions before the elimination of the Goldstone fields corresponding to the broken Lorentz transformations.

In section three we will consider the super p-brane. In this case we not only have diffeomorphism invariance, but also kappa transformations. The discussion follows a similar path to that given in section two for the bosonic case. Now the local subgroup will include the unbroken translations and also the unbroken supersymmetries. The corresponding local transformations will lead for the superpoint particle to world line diffeomorphisms and kappa supersymmetry transformations respectively. However, for the general super p-brane the same results will hold except that one must include an additional Lorentz rotation. The derivation of these results depend crucially on having Goldstone bosons associated with the broken Lorentz transformations. For the case of super p-branes we will also use the ideas of [15] and not introduce any superfields. In particular, we will take the fields to just depend on the parameters ξ and not on any additional Grassman odd parameters. As explained in reference [15], in a non-linear realisation all the fields are associated with generators of the algebras involved and so appear automatically in the group element with their corresponding generator.

2. The point particle and bosonic brane

We wish to construct the non-linear realisation that leads to dynamics of the bosonic p-brane moving in a D-dimensional space-time and so consider the algebra of translations and Lorentz rotations $ISO(1, D-1)$

$$[J_{\underline{ab}}, J_{\underline{cd}}] = -i\eta_{\underline{bc}}J_{\underline{ad}} + i\eta_{\underline{ac}}J_{\underline{bd}} + i\eta_{\underline{bd}}J_{\underline{ac}} - i\eta_{\underline{ad}}J_{\underline{bc}} \tag{2.1}$$

$$[J_{\underline{ab}}, P_{\underline{c}}] = -i\eta_{\underline{bc}}P_{\underline{a}} + i\eta_{\underline{ac}}P_{\underline{b}} . \tag{2.2}$$

In terms of our above discussion in the introduction $K = SO(1, D-1)$ and L contains the translations $P_{\underline{a}}$, $\underline{a} = 0, 1, \ldots, D-1$. We sub-divide the latter into $L_1 = P_a$, $a = 0, \ldots, p$ and $L_1 = P_{a'}$, $a' = p+1, \ldots, D-1$ and take the local sub-algebra of the non-linear realisation to be $H = ISO(1, p) \otimes SO(D-p-1)$ which contains the generators $P_a, J_{ab}, J_{a'b'}$. The P_a correspond to the unbroken translations and $J_{ab}, J_{a'b'}$ the unbroken Lorentz rotations. We are using the notation that an underlined index, i.e. $\underline{a}$ goes over all possible values from 0 to $D-1$ while the unprimed indices a take the values $a = 0, \ldots, p$ and primed indices a' take the values $p+1, \ldots, D-1$. The latter are the indices which are longitudinal and transverse to the brane respectively. The new feature is that we have placed the unbroken translations in the local sub-algebra of the non-linear realisation.

We now construct the non-linear realisation with algebra $G = ISO(1, D-1)$ with local sub-algebra $H = ISO(1, p) \otimes SO(D-p-1)$. We take the group element to be given by

$$g = e^{ix^a P_a} e^{ix^{a'} P_{a'}} e^{i\phi_a{}^{b'} J_a{}_{b'}} . \tag{2.3}$$

We note that this is not the most general line element as we have used the local Lorentz transformations to set part of the group element to one. All the above fields are taken to depend on the parameters ξ^i, $i = 0, \ldots, p$. As such the procedure has some aspects in common with the case of an internal symmetry. The non-linear realisation is by definition a theory that is invariant under the transformations of equation (1.3) which in this case reads

$$g(\xi) \rightarrow g_0 g(\xi), \quad \text{and} \quad g(\xi) \rightarrow g(\xi)h(\xi) . \tag{2.4}$$

We are particularly interested in local unbroken translations which are the new feature compared to other treatments of the bosonic brane and can be used to fix the field x^a that appears in the group element together with the unbroken translations P_a. As such we can consider local transformations of the form $h = e^{ir^a P_a}$ where r^a is an arbitrary function of ξ. Carrying out the transformation $g(\xi) \rightarrow g(\xi)h(\xi)$ we find that the fields transform as

$$\delta x^n = r^b(\Phi^{-1})_b{}^n \equiv s^n, \; \delta x^{n'} = r^b(\Phi^{-1})_b{}^{n'}, \; \delta\phi_a{}^{b'} = 0 \tag{2.5}$$

where for an arbitrary Lorentz transformation involving only the broken Lorentz generators $J^a{}_{b'}$ we define

$$e^{-i\phi \cdot J} P_{\underline{a}} e^{i\phi \cdot J} = \Phi_{\underline{a}}{}^{\underline{b}} P_{\underline{b}} . \tag{2.6}$$

It is convenient to use an explicit form of the Lorentz transformation $\Phi_{\underline{a}}{}^{\underline{b}}$. We may write it in the form

$$\Phi = \begin{pmatrix} I_1 & \varphi \\ -\varphi^T & I_2 \end{pmatrix} \begin{pmatrix} B_1 & 0 \\ 0 & B_2 \end{pmatrix} = \begin{pmatrix} B_1 & \varphi B_2 \\ -\varphi^T B_1 & B_2 \end{pmatrix} \tag{2.7}$$

where $B_1 = (I_1 + \varphi\varphi^T)^{-\frac{1}{2}}$ and $B_2 = (I_2 + \varphi^T\varphi)^{-\frac{1}{2}}$. The inverse Lorentz transformation is given by

$$\Phi^{-1} = \begin{pmatrix} B_1 & -B_1\varphi \\ B_2\varphi^T & B_2 \end{pmatrix} . \tag{2.8}$$

The broken Lorentz Goldstone fields $\phi_a{}^{b'}$ in (2.3) and $\varphi_a{}^{b'}$ are related by

$$(\varphi)_a{}^{b'} = \phi_a{}^{c'}\left(\frac{\sinh \tilde{V}}{\tilde{V}\cosh \tilde{V}}\right)_{c'}{}^{b'}, \qquad (\tilde{V}^2)_{a'}{}^{b'} \equiv -\phi^c{}_{a'}\phi_c{}^{b'}. \tag{2.9}$$

For more details on this particular parameterisation of the Lorentz transformation we refer the reader to reference [15]. In terms of this parameterisation the variations of equation (2.5) can be written as

$$\delta x^n = r^b(B_1)_b{}^n \equiv s^n, \ \delta x^{n'} = -r^c(B_1)_c{}^b\varphi_b{}^{n'} = -s^b\varphi_b{}^{n'} . \tag{2.10}$$

The dynamics is built out of the Cartan forms $\mathcal{V} = -ig^{-1}dg = \mathcal{V}_i d\xi^i$

$$\mathcal{V}_i = -ig^{-1}\partial_i g = e_i{}^a P_a + e_i{}^{a'} P_{a'} + \frac{1}{2}w_{i\underline{a}}{}^{\underline{b}}J^{\underline{a}}{}_{\underline{b}} \tag{2.11}$$

which are inert under rigid G transformations and transform under local H transformations as

$$\mathcal{V}_i \rightarrow h^{-1}\mathcal{V}_i h - ih^{-1}\partial_i h . \tag{2.12}$$

The Cartan forms are given by

$$e_i{}^a = (\partial_i x^c - \partial_i x^{b'}\varphi_{b'}^{Tc})(B_1)_c{}^a, \ e_i{}^{a'} = (\partial_i x^c\varphi_c{}^{b'} + \partial_i x^{b'})(B_2)_{b'}{}^{a'}, \ w_{i\underline{a}}{}^{\underline{b}} = (\Phi^{-1})_{\underline{a}}{}^{\underline{c}}\partial_i(\Phi)_{\underline{c}}{}^{\underline{b}} . \tag{2.13}$$

$e_i{}^{\underline{n}}$ is the vielbein and $w_{i\underline{a}}{}^{\underline{b}}$ is the spin connection. The transformations of the Cartan forms under the local transformations corresponding to P_a are given by

$$\delta e_i{}^a = \partial_i r^a - r^b w_{ib}{}^a, \quad \delta e_i{}^{a'} = -r^b w_{ib}{}^{a'}, \quad \delta w_{i\underline{a}}{}^{\underline{b}} = 0 . \tag{2.14}$$

2.1 Bosonic point particle

We now consider the case of the point particle, that is $p = 0$. Although this is a very elementary system it will allow us to illustrate how the world-line reparameterisation arises from the non-linear realisation by taking the unbroken time translation to be part of the local subgroup. To have a more usual notation we take $\tau = \xi^0$ and $t = x^0$. The Cartan forms of equation (2.13) are given by

$$e \equiv e_0{}^0 = \left(\frac{dt}{d\tau} - \frac{dx^{b'}}{d\tau}\varphi_{b'}\right)(1 + \varphi^{b'}\varphi_{b'})^{-\frac{1}{2}}, \ e^{a'} \equiv e_0{}^{a'} = \left(\frac{dt}{d\tau}\varphi^{b'} + \frac{dx^{b'}}{d\tau}\right)(B_2)_{b'}{}^{a'},$$

$$w_{\underline{a}}{}^{\underline{b}} \equiv w_{0\underline{a}}{}^{\underline{b}} = (\Phi^{-1})_{\underline{a}}{}^{\underline{c}}\frac{d}{d\tau}(\Phi)_{\underline{c}}{}^{\underline{b}} \tag{2.15}$$

where $\varphi_0{}^{b'} \equiv \varphi^{b'}, \varphi^0{}_{b'} \equiv \varphi_{b'} = -\varphi^{b'}$. Under a local P_0 transformation the fields transform as

$$\delta t = r(1 + \varphi^{b'}\varphi_{b'})^{-\frac{1}{2}} \equiv s, \quad \delta x^{n'} = -s\varphi^{n'}, \tag{2.16}$$

where $r = r^0$ and $s = s^0$, while the Cartan forms transform associated with the translations as

$$\delta e = \frac{dr}{d\tau}, \quad \delta e^{a'} = -rw_0{}^{a'}, \quad \delta w_{\underline{a}}{}^{\underline{b}} = 0. \tag{2.17}$$

Examining the first of equations in (2.17) we realise that an action invariant under local P_0 transformations, and indeed all the transformations of equation (2.4), is given by

$$A = -\int d\tau \, e = -\int d\tau \frac{(\frac{dt}{d\tau} - \frac{dx^{b'}}{d\tau}\varphi_{b'})}{(1 + \varphi^{b'}\varphi_{b'})^{\frac{1}{2}}} \; . \tag{2.18}$$

Varying this action we find the equation of motion for $\varphi^{a'}$ implies that

$$\varphi^{a'} = -\frac{dx^{a'}}{d\tau} \Big/ \frac{dt}{d\tau} \; . \tag{2.19}$$

Substituting this algebraic equation back into the action we find the standard action for the point particle, namely

$$A = -\int d\tau \left(-\frac{dx^{\underline{n}}}{d\tau}\frac{dx_{\underline{n}}}{d\tau}\right)^{\frac{1}{2}} \; . \tag{2.20}$$

Taking the equation of motion of $\varphi^{a'}$, that is implementing equation (2.19), corresponds to setting the Cartan form $e^{a'}$ of equation (2.15) to zero, i.e.

$$e^{a'} = 0 \; . \tag{2.21}$$

We see, using equation (2.17), that this condition is not invariant under the transformations of the non-linear realisation, but instead implies the equation of motion of $x^{\underline{a}}$, $w_0{}^{a'} = 0$, for the point particle. That it means an equation of motion follows from the fact that $e^{a'} = 0$ is a consequence of the above invariant action and so all the equations of motion of the action must rotate into each other. We note that if one does not take the local sub-algebra to contain the unbroken translations, then setting the Cartan form $e^{a'} = 0$ is an invariant condition and so is an example of the inverse Higgs effect. This latter point of view and the above action are the subject of reference [14].

Finally, we examine the transformations of the fields if equation (2.19) holds. The transformation of t of equation (2.16) can be considered as a transformation of τ by identifying $\delta t = s = \frac{dt}{d\tau}\rho$ where $\delta\tau = \rho$. Using equations (2.16), (2.17) and (2.19) we find that

$$\delta x^{n'} = \frac{dx^{n'}}{d\tau}\rho, \quad \delta e = \frac{d}{d\tau}(e\rho) \tag{2.22}$$

which we recognise as the standard variations under a diffeomorphism in τ. In fact even before one implements $e^{a'} = 0$, one finds that the transformations in (2.16) differ from those of equation (2.22) by anti-symmetric combinations of equations of motion. We have therefore found that implementing the $\varphi^{a'}$ equation of motion converts the local P_0 transformation of the non-linear realisation into a diffeomorphism of the world-line.

2.2 The bosonic p-brane

We now consider the bosonic p-brane which we will see requires an additional step compared to that for the point particle. An action that has been derived from the theory of non-linear realisations in the absence of local unbroken translation and depends on all the Goldstone bosons, including those for broken Lorentz transformations is given by [15]

$$\mathcal{L} = -\frac{1}{(p+1)!}\epsilon_{a_0,...,a_p}e^{a_0} \wedge .... \wedge e^{a_p} = d^{p+1}\xi \det e_i{}^a .\tag{2.23}$$

Here $e^a = d\xi^i e_i{}^a$ is the Cartan form associated to P_a of equation (2.13).

This action is a functional of $\varphi_a{}^{n'}$ and $x^{\underline{n}}$ and varying with respect to $\varphi_a{}^{n'}$ leads to an equation of motion that implies

$$\varphi_a{}^{n'} = -\frac{\partial X^{n'}}{\partial\xi^i}\left(\frac{\partial X^a}{\partial\xi^i}\right)^{-1} = -\frac{\partial X^{n'}}{\partial\xi^i}(e^{-1})_a{}^i\tag{2.24}$$

which is the solution of $e_i{}^{a'} = 0$ [15]. Eliminating $\varphi_a{}^{n'}$ from the action we find the usual action for the p-brane. Actions for branes including supplementary variables, Lorentz vector harmonics, were considered in [18]. These actions were not derived from the view point of non-linear realizations and contain redundant fields that satisfy constraint equations.

The action (2.23) is constructed from the Cartan forms of equation (2.11) and it is invariant under local $SO(1,p) \otimes SO(D-p-1)$ transformations. Consequently, it is also invariant under rigid $SO(1,D-1)$ transformations. However, under the local transformations corresponding to local P_a transformations of equation (2.14) this action transforms as

$$\delta\int d^{p+1}\xi \det e_i{}^a = \int d^{p+1}\xi(\det e_i{}^a) \, r^k(e_k{}^{b'}w_{ib'}{}^a(e^{-1})_a{}^i - (e^{-1})_a{}^i e_i{}^{b'}w_{kb'}{}^a)\tag{2.25}$$

where $r^i = r^a(e^{-1})_a{}^i$. Although this does not vanish in general it does vanish when $e_i{}^{a'} = 0$ and since this is the equation of motion of $\varphi_a{}^{n'}$ it follows that one can cancel this term by adding a suitable quantity to the variation of $\varphi_a{}^{n'}$. Under a combined local translations with parameter r^a and the above local Lorentz transformation with parameter $r_a{}^{b'}$ we find that one form $e^{\underline{a}}$ varies as

$$\delta e^a = dr^a + w^a{}_b r^b + e^{b'}r_{b'}{}^a, \qquad \delta e^{a'} = e^b r_b{}^{a'} + w^{a'}{}_b r^b.\tag{2.26}$$

One can verify that the Lorentz transformation which indeed cancels the above variation of the action (2.25) is given by

$$r^{a'}{}_b = -(e^{-1})_c{}^i r^c w_{ib}{}^{a'} + r_b((e^{-1})_c{}^i w_i{}^{ca'}).\tag{2.27}$$

As we have an invariance of the action the equation of motion $e_i{}^{a'} = 0$ is preserved provided we use the other equations of motion, as must be the case.

It is instructive to consider the transformation of $e_i{}^a$ under a local translation. Using $r^i = (e^{-1})_a{}^i r^a$ we can express equation (2.14) as

$$\delta e_i{}^a = \partial_i(r^j)e_j{}^a + r^j(\partial_i e_j{}^a - w_{ib}{}^a e_i{}^b)$$

$$= \partial_i(r^j)e_j{}^a + r^j\partial_j e_i{}^a + (r^j w_j{}^a{}_b)e_i{}^b + r^j(w_{ib'}{}^a e_j{}^{b'} - w_{jb'}{}^a e_i{}^{b'}) \qquad (2.28)$$

where in the last step we have used the Maurer-Cartan equations. Thus we find the standard diffeomorphism plus a local Lorentz rotation plus a term that vanishes if $e_i{}^{a'} = 0$. Furthermore if we consider a local translation combined with the Lorentz transformation of (2.26) we find that the variation of $e_i{}^a$ is given by

$$\delta e_i{}^a = \partial_i(r^j)e_j{}^a + r^j\partial_j e_i{}^a + (r^j w_j{}^a{}_b)e_i{}^b + r^j(w_{ib'}{}^a e_j{}^{b'} - w_{jb'}{}^a e_i{}^{b'}) + e_i{}^{b'} r_{b'}{}^a. \qquad (2.29)$$

We note that the first two terms are a diffeomorphism, the third term is a local $SO(1,p)$ rotation and the remaining terms vanish when (2.24) is used.

Hence we have found an invariant action which is constructed from the Cartan forms, but we have had to modify the local transformations from those that follow from the strict application of the non-linear realisation. The theory is invariant under a left transformation of the form

$$h = \exp(ir^a P_a + ir_a{}^{b'} J^a{}_{b'}) \qquad (2.30)$$

where $r^{a'}{}_b$ is given in equation (2.27).

3. The Super Point Particle and Super Brane

In this section we generalize the discussion of the last section on the bosonic particle and p-brane to the case of supersymmetric branes. In particular, we will study how the kappa symmetry of supersymmetric brane actions arise as local transformations when we take the unbroken supersymmetry generators as part of the local sub-algebra of the non-linear realisation. That kappa transformations might be part of the non-linear realisation has been suggested in the past. For example, in reference [19], the idea of incorporating local unbroken supersymmetry transformations was considered and some progress was made to deriving κ-transformations as part of the non-linear realisation. However, the author considered the theory to be defined on the coset and did not introduce Goldstone bosons associated with the broken Lorentz transformations and so was able to obtain only partial results.

We take the algebra G of the non-linear realisation to be the Super Poincare algebra which contains in addition to equations (2.1) and (2.2) the commutation relations

$$\left[Q_{\underline{\alpha}},\ J_{\underline{ab}}\right] = -\frac{i}{2}(\Gamma_{\underline{ab}})_{\underline{\alpha}}{}^{\underline{\beta}}Q_{\underline{\beta}}, \quad \left[Q_{\underline{\alpha}},\ P_{\underline{a}}\right] = 0, \quad \{Q_{\underline{\alpha}}, Q_{\underline{\beta}}\} = 2(\Gamma^{\underline{a}}C^{-1})_{\underline{\alpha}\underline{\beta}}\ P_{\underline{a}}. \qquad (3.1)$$

In order to construct the non-linear realisation we must choose a local sub-algebra. We take this to have a Grassman even part given by the Lorentz transformations $SO(1,p) \otimes SO(D-p-1)$ and the local unbroken translations P_a while the Grassman odd part contains the supercharges Q^* which are related to the full set of supersymmetry generators Q by a

projection operator $\mathcal{P}$ as

$$Q^*_{\underline{\alpha}} = \mathcal{P}_{\underline{\alpha}}{}^{\underline{\beta}} Q_{\underline{\beta}}. \tag{3.2}$$

For the sake of simplicity we will study the case where half of the supercharges are unbroken. The non-zero components of $Q^*_{\underline{\alpha}}$ are often denoted by Q_α in the literature. The projector must be such that the chosen algebra must close and in particular

$$\{Q^*_{\underline{\alpha}}, Q^*_{\underline{\beta}}\} = (\mathcal{P}\Gamma^a C^{-1} \mathcal{P}^T)_{\underline{\alpha}\,\underline{\beta}} P_a. \tag{3.3}$$

We write the projection operator $\mathcal{P}$ as

$$\mathcal{P} = \frac{1}{2}(1 + \Gamma_*), \qquad \Gamma_*{}^2 = 1, \tag{3.4}$$

(3.3) requires

$$\Gamma^a C^{-1} \Gamma_*{}^T C = \Gamma_* \Gamma^a, \qquad \Gamma^{a'} C^{-1} \Gamma_*{}^T C = -\Gamma_* \Gamma^{a'}. \tag{3.5}$$

For the superparticle, the supercharges are IIA spinors, and Γ_* is , up to sign, as

$$\Gamma_* := \Gamma^0 \Gamma_{11}. \tag{3.6}$$

For the super p-branes the supercharges are minimal spinors and they exist for $p = 1, 2$ mod 4 [20]. The Γ_* is , up to sign, as

$$\Gamma_* := \frac{1}{(p+1)!} \epsilon^{a_0 \dots a_p} \Gamma_{a_0} \dots \Gamma_{a_p}. \tag{3.7}$$

The formulation being considered in this paper applies equally well to branes which preserve some supersymmetry and those that break all the supersymmetries. In the latter case there are no unbroken supercharges and so no corresponding supercharges in the local subalgebra of the non-linear realisation. This is consistent with the fact that the branes that break all the supersymmetries have no kappa invariance.

We may then write the group elements in the form

$$g = g_0\, U, \qquad g_0 := e^{ix^{\underline{a}} P_{\underline{a}}}\, e^{\bar{\theta}^{\underline{\alpha}} Q_{\underline{\alpha}}}, \qquad U := e^{i\phi_a{}^{b'} J^a{}_{b'}}, \tag{3.8}$$

where $\bar{\theta}^{\underline{\alpha}} = \theta_{\underline{\beta}} C^{\underline{\beta}\underline{\alpha}}$ or equivalently $\bar{\theta} = \theta^T C$. We parametrise the group element g, and so the fields $x^{\underline{a}}$, $\theta_{\underline{\alpha}}$ and $\phi_a{}^{b'}$, to depend on the worldvolume Grassman even parameters $\xi^i, i = 0, 1, \dots, p$ [15]. We have used the local symmetry under the unbroken local Lorentz transformations to bring the group element into the above form, however, we have not used the local transformations involving the unbroken translations or supersymmetries to fix any of the corresponding fields. The above differs from the usual treatment where one usually takes the fields to depend on the parameters ξ^i as well as a set of Grassman odd variables θ_α. As explained in [15] the use of Goldstone superfields to describe the dynamics of the superbranes is not needed and leads to a redundancy.

The Cartan forms are given by

$$\mathcal{V} = U^{-1} \mathcal{V}_0 U - i U^{-1} dU = e^{\underline{a}} P_{\underline{a}} - i \bar{e}^{\underline{\alpha}} Q_{\underline{\alpha}} + \frac{1}{2} w^{\underline{ab}} J_{\underline{ab}}. \tag{3.9}$$

The expression $\mathcal{V}_0$ which appears in the first term is given by

$$\mathcal{V}_0 = -ig_0^{-1}dg_0 = \pi^{\underline{a}}P_{\underline{a}} - i\bar{\pi}^{\underline{\alpha}}Q_{\underline{\alpha}} \tag{3.10}$$

where $\pi^{\underline{a}}$, $\bar{\pi}^{\underline{\alpha}}$ are supervielbeins of Poincare superspace which are given by

$$\pi^{\underline{a}} = dx^{\underline{a}} + i\bar{\theta}\Gamma^{\underline{a}}d\theta, \qquad \bar{\pi}^{\underline{\alpha}} = d\bar{\theta}^{\underline{\alpha}}. \tag{3.11}$$

Using equation (2.6) and the analogous equation for the spinor representation $\tilde{\Phi}_\beta{}^\alpha$ of the boost U,

$$U^{-1}P_{\underline{a}}U = \Phi_{\underline{a}}{}^{\underline{b}}P_{\underline{b}}, \qquad U^{-1}Q_{\underline{\alpha}}U = \tilde{\Phi}_{\underline{\alpha}}{}^{\underline{\beta}}Q_{\underline{\beta}}, \tag{3.12}$$

we find that

$$e^{\underline{a}} = \pi^{\underline{b}}\Phi_{\underline{b}}{}^{\underline{a}}, \qquad \bar{e}^{\underline{\alpha}} = \bar{\pi}^{\underline{\beta}}\tilde{\Phi}_{\underline{\beta}}{}^{\underline{\alpha}}, \qquad w_{\underline{a}}{}^{\underline{b}} = (\Phi^{-1})_{\underline{a}}{}^{\underline{c}}d\Phi_{\underline{c}}{}^{\underline{b}}, \tag{3.13}$$

where $\Phi_{\underline{b}}{}^{\underline{a}}$ is given in equation (2.7) and

$$\tilde{\Phi}_{\underline{\alpha}}{}^{\underline{\beta}} = \left(\exp(\frac{i}{2}\phi \cdot J)\right)_{\underline{\alpha}}{}^{\underline{\beta}} = \left(\exp(i\phi_a{}^{b'}J^a{}_{b'})\right)_{\underline{\alpha}}{}^{\underline{\beta}} = \left(\exp(\frac{1}{2}\phi_a{}^{b'}\Gamma^a\Gamma_{b'})\right)_{\underline{\alpha}}{}^{\underline{\beta}}. \tag{3.14}$$

The relationship between the spinorial and the vectorial representations is as usual given by

$$\tilde{\Phi}\Gamma_{\underline{a}}\tilde{\Phi}^{-1} = (\Phi^{-1})_{\underline{a}}{}^{\underline{b}}\Gamma_{\underline{b}}. \tag{3.15}$$

We note that the expression for $e^{\underline{a}}$ is the same as that of equation (2.13) except that $dx^{\underline{a}}$ is replaced by $\pi^{\underline{a}}$. The transformations of the Cartan forms under the local transformations corresponding to P_a, Q^* and $J^a{}_{b'}$ are given by

$$\delta e^a = dr^a + w^a{}_b r^b + e^{b'}r_{b'}{}^a + 2i(\bar{\kappa}_*\Gamma^a e), \qquad \delta e^{a'} = e^b r_b{}^{a'} + w^{a'}{}_b r^b + 2i(\bar{\kappa}_*\Gamma^{a'}e), \tag{3.16}$$

$$\delta\bar{e}^{\underline{\alpha}} = D\bar{\kappa}_*^{\underline{\alpha}} + \frac{1}{2}\bar{e}^{\underline{\beta}}(\Gamma_{ab'}r^{ab'})_{\underline{\beta}}{}^{\underline{\alpha}}, \qquad \delta w^{ab} = w^{ac'}r_{c'}{}^b - w^{bc'}r_{c'}{}^a,$$

$$\delta w^{ab'} = dr^{ab'} + w^{ac}r_c{}^{b'} - w^{b'c'}r_{c'}{}^a, \qquad \delta w^{a'b'} = w^{a'c}r_c{}^{b'} - w^{b'c}r_c{}^{a'}, \tag{3.17}$$

where $\bar{\kappa}_* = \bar{\kappa}_*\mathcal{P}$ is the Q^* transformation parameter and $D\bar{\kappa}_* = d\bar{\kappa}_* - \frac{1}{4}\bar{\kappa}_*(\Gamma_{\underline{ab}}w^{\underline{ab}})$ is the Lorentz covariant derivative.

As in the bosonic case it will be much easier to treat the case of the superparticle which is straightforward, while the super p-brane has an additional step beyond those that arise naturally in the theory of non-linear realisations. As such we first consider the case of the superparticle.

3.1 Superparticle

For this case the local sub-algebra consists of the generators of SO(D-1), the translation P_0, and half of the supercharges Q^*. The field in the group element of equation (3.8) just depend on one parameter which we take to be τ. Using equation (2.7) in equation (3.13), the Cartan forms, for the superparticle are given by

$$e^0 = (\pi^0 - \pi^{b'}\varphi_{b'})(1 + \varphi^{b'}\varphi_{b'})^{-\frac{1}{2}}, \quad e^{a'} = (\pi^0\varphi^{b'} + \pi^{b'})(B_2)_{b'}{}^{a'}, \tag{3.18}$$

and the remaining Cartan forms $\bar{e}^{\underline{\alpha}}$ and $w_{\underline{a}}{}^{\underline{b}}$ can also be read off from (3.13).

Let us first consider the effect of carrying out a local transformation for the P_0 translation. Taking $h = e^{irP_0}$ the fields transform as

$$\delta x^{\underline{a}} = r(\Phi^{-1})_0{}^{\underline{a}}, \qquad \delta\bar{\theta}^{\underline{\alpha}} = \delta\phi_0{}^{b'} = 0, \tag{3.19}$$

which is given more explicitly by,

$$\delta x^0 = r(1 + \varphi^{b'}\varphi_{b'})^{-\frac{1}{2}} \equiv s, \qquad \delta x^{a'} = -s\varphi^{a'}, \tag{3.20}$$

while the Cartan forms transform as

$$\delta e^0 = dr, \quad \delta e^{a'} = -rw_0{}^{a'}, \quad \delta w_{\underline{a}}{}^{\underline{b}} = 0, \quad \delta\bar{e}^{\underline{\alpha}} = 0. \tag{3.21}$$

Next we consider the local transformations generated by the unbroken supersymmetries. The local transformation is given by taking $h = e^{\bar{\kappa}_* Q^*}$, where the $\bar{\kappa}_*^{\underline{\alpha}} = (\bar{\kappa}_* \mathcal{P})^{\underline{\alpha}}$ are 16 independent gauge parameters that depend on τ. The transformations of the fields are given by

$$\delta x^{\underline{a}} = i\delta\bar{\theta}\Gamma^{\underline{a}}\theta, \qquad \delta\bar{\theta}^{\underline{\alpha}} = \bar{\kappa}_*^{\underline{\beta}}\tilde{\Phi}_{\underline{\beta}}^{-1\underline{\alpha}}, \qquad \delta\phi_0{}^{b'} = 0. \tag{3.22}$$

The corresponding transformations of the Cartan forms are

$$\delta e^{\underline{a}} = 2i(\bar{\kappa}_*\Gamma^{\underline{a}}e), \qquad \delta\bar{e}^{\underline{\alpha}} = (D\bar{\kappa}_*)^{\underline{\alpha}}, \qquad \delta w_{\underline{a}}{}^{\underline{b}} = 0. \tag{3.23}$$

The Cartan forms are inert under the rigid transformations and only transform under the local part of the non-linear realisation and so to construct an action out of the Cartan forms we need only take into account of the effect of the local subgroup. Let us first construct an action invariant under local P_0 translation and local Lorentz transformations. One such term in the action is the one form e^0, as in the purely bosonic particle case. The presence of an additional term is due to the existence of a non-trivial Chevalley Eilenberg cohomology of the Super Poincare group [21]. In particular, there exists a closed two form $-i\bar{e}^{\underline{\alpha}}\bar{e}^{\underline{\beta}}(\Gamma_{11}C^{-1})_{\underline{\alpha\beta}}$ that is invariant under local P_0 transformations and can be written as

$$-i\bar{e}^{\underline{\alpha}} \wedge \bar{e}^{\underline{\beta}}(\Gamma_{11}C^{-1})_{\underline{\alpha\beta}} = id\bar{\theta}\Gamma_{11}d\theta = id(\bar{\theta}\Gamma_{11}d\theta). \tag{3.24}$$

It follows that $i\bar{\theta}\Gamma_{11}d\theta$ must transform as a d of something under a local P_0 transformation.

Hence, assuming the action depends only on velocities, we may take for our action

$$A = -\int (\mathcal{L}_{NG} + b\mathcal{L}_{WZ}) = -\int \left(e^0 + i\,b\,\bar{\theta}\Gamma_{11}d\theta\right). \tag{3.25}$$

The b is an arbitrary real constant that is not fixed by demanding invariance under the unbroken local Lorentz and P_0 translations. It is fixed once the invariance under the local unbroken supersymmetry is imposed. Using (3.22) and (3.23) the variation of the lagrangian under these transformation is given by

$$\delta_\kappa \mathcal{L} = 2i(\bar{\kappa}_*(1 + b\Gamma^0\Gamma_{11})\Gamma^0 e) + \text{surface term}. \tag{3.26}$$

Using Γ_* in (3.6) we see that the action (3.23) is invariant under local unbroken supersymmetries when $b = -1$. It is also true for $b = 1$ if we choose $\Gamma_* = -\Gamma^0\Gamma_{11}$ in (3.6).

The relation of the action of equation (3.25) with ordinary superparticle action is obtained by considering the equations of motion for $\varphi^{a'}$ [15] which implies

$$\varphi^{a'} = -\frac{\pi_0{}^{a'}}{\pi_0{}^0}, \qquad \pi_0{}^{\underline{a}} \equiv \dot{x}^{\underline{a}} + i\bar{\theta}\Gamma^{\underline{a}}\dot{\theta}. \tag{3.27}$$

Substituting this algebraic equation back into the action we find the standard action for the point particle, namely

$$A = -\int d\tau \left(\sqrt{-\pi_0{}^{\underline{a}}\pi_0{}^{\underline{b}}\eta_{\underline{ab}}} - i\bar{\theta}\Gamma_{11}\dot{\theta} \right). \tag{3.28}$$

We will also see now that the kappa symmetry discussed above coincides with kappa matrix known in the literature once we eliminate the non-dynamical Goldstone fields $\varphi_0{}^{b'}$. We rewrite κ_* in terms of the 32 component spinor κ

$$\bar{\kappa}_* = \bar{\kappa}\frac{1}{2}(1 + \Gamma^0\Gamma_{11}). \tag{3.29}$$

Note that when we work with κ instead of κ_* we introduce the reducibility of kappa transformation. This situation is similar to the non-relativistic branes case [22] where the kappa symmetry is written in terms of the analogous of κ_* and it is therefore irreducible. Let us introduce the field dependent gamma matrix

$$\Gamma_\kappa(\varphi) = \tilde{\Phi}(\varphi)\Gamma_0\Gamma_{11}\tilde{\Phi}(\varphi)^{-1} = \Phi_0{}^{\underline{a}}\Gamma_{\underline{a}}\Gamma_{11}. \tag{3.30}$$

It is trivial to verify $\Gamma_\kappa{}^2 = 1$. The transformation of equation (3.22) is then given by

$$\delta\bar{\theta} = \bar{\kappa}\frac{1}{2}(1 + \Gamma^0\Gamma_{11})\tilde{\Phi}^{-1} = \bar{\tilde{\kappa}}\frac{1}{2}(1 - \Gamma_\kappa(\varphi)), \qquad \tilde{\kappa} \equiv \tilde{\Phi}\kappa. \tag{3.31}$$

Once we eliminate $\varphi^{b'}$ we find that

$$\Gamma_\kappa(x, \theta) = \frac{\pi_0{}^{\underline{a}}\Gamma_{\underline{a}}\Gamma_{11}}{\sqrt{-\pi_0{}^{\underline{b}}\pi_0{}^{\underline{c}}\eta_{\underline{bc}}}} \tag{3.32}$$

which is the well known expression of the Γ_κ matrix.

Summing up, the action, with the lowest number of derivatives, for the super point particle is uniquely determined by the non-linear realisations once we take the local subalgebra to include the unbroken supersymmetries and translations. Furthermore we have seen that the local transformations for the unbroken supersymmetries reduces to the usual expression for kappa transformations once we have eliminated the non-dynamical Goldstone fields corresponding to the broken Lorentz transformations. However, the latter fields do play a crucial role in the construction.

Here we do not discuss the world line diffeomorphism explicitly. One can prove along the lines of previous section that the diffeomorphism is equivalent to the P_0 transformation combined with a Q_* transformation.

3.2 Super p-brane

We have seen in the bosonic case that the local translations alone are not an invariance of the p-brane action and we need to modify them by adding a local Lorentz transformation whose parameters are not independent of the parameters of the local translations. As we will now see an analogous situation occurs for the unbroken supersymmetry transformations of the supersymmetric branes . The transformations of the Cartan forms under the local transformations corresponding to P_a, Q^* and $J^a{}_{b'}$ are given in (3.16) and (3.17).

An action of super p-branes which was constructed from the theory of non-linear realisation but not taking the unbroken translations and supersymmetries in the local subalgebra was given by [15]

$$A = -T \int (\mathcal{L}^{NG} + b \, \mathcal{L}^{WZ}) \tag{3.33}$$

where

$$\mathcal{L}^{NG} = -\frac{1}{(p+1)!} \epsilon_{a_0, \dots, a_p} e^{a_0} \wedge \dots \wedge e^{a_p}, \tag{3.34}$$

and the WZ action [23] is given by

$$\mathcal{L}^{WZ} = \frac{-1}{(p+1)!} \sum_{r=0}^{p} (-1)^r \begin{pmatrix} p+1 \\ r+1 \end{pmatrix} \pi^{\underline{a}_p} \wedge \dots \wedge \pi^{\underline{a}_{r+1}} \wedge K^{\underline{a}_r} \wedge \dots \wedge K^{\underline{a}_1} \wedge K_{\underline{a}_1, \dots \underline{a}_p}, \tag{3.35}$$

where $\begin{pmatrix} p+1 \\ r+1 \end{pmatrix}$ is the binomial coefficient and

$$K^{\underline{a}} = i\bar{\theta}\Gamma^{\underline{a}}d\theta, \qquad K_{\underline{a}_1, \dots \underline{a}_p} = i\bar{\theta}\Gamma_{\underline{a}_1, \dots \underline{a}_p}d\theta . \tag{3.36}$$

The action depends on all the Goldstone bosons of the theory including those corresponding to the broken Lorentz transformation, namely $\phi_a{}^{b'}$. However, the WZ term $\mathcal{L}^{WZ}$ does not depend on these latter fields .

Like in the case of the superparticle let us assume that our brane breaks half of the supersymmetries and there remain the unbroken supersymmetry generators $Q^*_{\underline{\alpha}}$. If we consider the unbroken generators as elements of the local algebra H, it can be seen that there is no action of the type equation (3.33) that is invariant under the right transformations generate by Q^*. As in the diffeomorphism of the bosonic p-brane we should also introduce local transformation associated to the broken Lorentz transformations. Let us consider the right action on the coset given by

$$h = e^{\bar{\kappa}_* Q^* + i r_a{}^{b'} J^a{}_{b'}} \tag{3.37}$$

where the κ_* are 16 independent spinor gauge parameters that depend on ξ and $r_a{}^{b'}$ are dependent gauge parameters linearly related to κ_*. They are the supersymmetric analogous of the ones appearing in the right action of the unbroken translation (2.30).

The gauge transformations on the fields are given by given

$$\delta x^{\underline{a}} = i\delta\bar{\theta}\Gamma^{\underline{a}}\theta, \qquad \delta\bar{\theta}^{\underline{\alpha}} = \bar{\kappa}_*^{\underline{\beta}} \tilde{\Phi}_{\underline{\beta}}^{-1\underline{\alpha}}, \qquad \delta\phi^{aa'} = r^{bb'} \left(\frac{W}{\sinh W} \right)_{bb'}{}^{aa'} \tag{3.38}$$

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where $r_{b'}{}^a = (\Phi^{-1}\delta\Phi)_{b'}{}^a$ is analogous of the components of the spin connection $w_{b'}{}^a$ in which the differential d is replaced by the variation δ. W is defined in the appendix of [15] by

$$(W^2)_{aa'}{}^{bb'} = -(\phi_a{}^{c'}\phi^b{}_{c'})\delta_{a'}{}^{b'} - \delta_a{}^b(\phi^c{}_{a'}\phi_c{}^{b'}) + 2\phi_a{}^{b'}\phi^b{}_{a'}. \tag{3.39}$$

The relation of $r_a{}^{b'}$ and κ_* will be determined by requiring the invariance of the lagrangian under (3.37). We can compute the variation of the lagrangian under this transformation

$$\delta_\kappa\mathcal{L}^{NG} = \frac{-1}{p!}\epsilon_{a_0...a_p}e^{a_0}...e^{a_{p-1}}(2i(\bar{\kappa}_*\Gamma^{a_p}e) + e^{b'}r_{b'}{}^{a_p}), \tag{3.40}$$

$$\delta_\kappa\mathcal{L}^{WZ} = \frac{(-1)^p}{p!}e^{\underline{a_0}}...e^{\underline{a_{p-1}}}(2i\,\bar{\kappa}_*\Gamma_{\underline{a_0}...\underline{a_{p-1}}}e) + \text{surface term}. \tag{3.41}$$

$\delta_\kappa\mathcal{L}^{WZ}$ is separated into two terms, the first one includes sum of terms with only longitudinal indices a_j's. Using Γ_* in (3.7) and

$$\Gamma_{a_0...a_{p-1}} = -\epsilon_{a_0...a_{p-1}a_p}\Gamma_*\Gamma^{a_p}, \tag{3.42}$$

the sum of the first terms of the $\delta_\kappa\mathcal{L}^{WZ}$ and the $\delta_\kappa\mathcal{L}^{NG}$ becomes

$$\delta_\kappa\mathcal{L}_1 := \frac{-2i}{p!}\epsilon_{a_0...a_p}e^{a_0}...e^{a_{p-1}}\left(\bar{\kappa}_*(1 + b(-1)^p\Gamma_*)\Gamma^{a_p}e\right). \tag{3.43}$$

Since $\bar{\kappa}_* = \bar{\kappa}_*\frac{1}{2}(1 + \Gamma_*)$ (3.43) vanishes when $b = (-1)^{p+1}$. (If we were taken the opposite sign choice for Γ_*, the lagrangian with $b = (-1)^p$ is invariant.) The remaining terms in the variation of the lagrangian are

$$\delta_\kappa\mathcal{L}_2 := \frac{-1}{p!}\epsilon_{a_0...a_p}e^{a_0}...e^{a_{p-1}}e^{b'}r_{b'}{}^{a_p} + b\frac{(-1)^p}{p!}\sum{}'e^{\underline{a_0}}...e^{\underline{a_{p-1}}}(2i\bar{\kappa}_*\Gamma_{\underline{a_0}...\underline{a_{p-1}}}e). \tag{3.44}$$

Here the sum $\sum'$ does not include terms with only longitudinal indices a_j's thus at least one of $\underline{a}_j$ is transverse a'_j. Since the world volume vielbein $e_i{}^a$ is non-singular we can determine $r_{b'}{}^{a_p}$ in terms of κ_* from $\delta_\kappa\mathcal{L}_2 = 0$. Thus the kappa transformation of the Goldstone fields $\varphi_b{}^{a'}$ is determined from (3.44). The total lagrangian is pseudo-invariant under the kappa transformation.

Let us see now that the kappa symmetry discussed above coincides with kappa transformation known in the literature once we eliminate the non-dynamical Goldstone fields $\varphi_a{}^{b'}$. We write $\bar{\kappa}_* = \bar{\kappa}\mathcal{P}$ in term of $\bar{\kappa}$ spinor with independent components

$$\bar{\kappa}_* = \bar{\kappa}\frac{1}{2}(1 + \Gamma_*). \tag{3.45}$$

In terms of this kappa parameter the transformation (3.38) is given by

$$\delta\bar{\theta} = \bar{\kappa}\frac{1}{2}(1 + \Gamma_*)\tilde{\Phi}^{-1} = \bar{\tilde{\kappa}}\frac{1}{2}(1 + \Gamma_\kappa), \qquad \tilde{\kappa} \equiv \tilde{\Phi}\kappa, \tag{3.46}$$

where

$$\Gamma_\kappa(\varphi) = \tilde{\Phi}\Gamma_*\tilde{\Phi}^{-1} = \frac{1}{(p+1)!}\epsilon^{a_0...a_p}\Phi_{a_0}^{-1\underline{b_0}}...\Phi_{a_p}^{-1\underline{b_p}}\Gamma_{\underline{b_0}}...\Gamma_{\underline{b_p}} \tag{3.47}$$

that verifies $\Gamma_\kappa{}^2 = 1$. Once we eliminate $\varphi_a{}^{b'}$ we get

$$\Gamma_\kappa(x, \theta) = \frac{1}{(p+1)!\sqrt{-\det G_{ij}}} \epsilon^{i_0 \dots i_p} \pi_{i_0}{}^{\underline{b}_0} \dots \pi_{i_p}{}^{\underline{b}_p} \Gamma_{\underline{b}_0} \dots \Gamma_{\underline{b}_p}. \qquad (3.48)$$

which is the well known expression of the Γ_κ matrix [24−26].

Summing up, for the case of the super p-brane we have seen that it is not enough to consider the unbroken supersymmetries in the local subgroup of the non-linear realisation in order to have the invariance of the super-brane action. We need to also carry out a local transformation associated with the Lorentz broken generator with the parameters $r_{b'}{}^a$ as functions of κ_*.

4. Discussion

In this paper we have derived the dynamics of bosonic and superbranes using a theory of non-linear realisations in which the unbroken translations and supercharges are part of the respective local sub-algebras. For the bosonic point particle and the superpoint particle this has led to a group theoretic derivation of world line reparameterisation invariance and kappa symmetry respectively. However, for the bosonic p-brane and super p-brane we have had to supplement the local transformations by a field dependent broken Lorentz transformation. The latter step is outside of the non-linear realisation as formulated in this paper. It might seem as if it were a compensating transformation, however, as formulated in this paper the unbroken Lorentz transformations have no local symmetry associated with them. It may be that there is a more general formulation of the non-linear realisation in which this field dependent broken Lorentz transformation is part of the non-linear realisation.

One of the aims of this paper is to further develop the theory of non-linear realisations in the context of branes so that one might apply it to the conjectures in reference [27] concerning brane dynamics and the Kac-Moody algebra E_{11}.

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